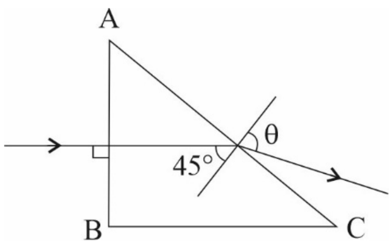
Solutions to JEE Advanced Home Practice Test -1 |JEE 2024 | Paper-2

PHYSICS

SINGLE DIGIT INTEGER TYPE

1.(2) $\mu \sin 45° = \sin\theta$

$\therefore \quad \sin\theta = \dfrac{1+0.4t}{\sqrt{2}} = \dfrac{1.4}{\sqrt{2}}$ at $t = 1$ sec



Differentiating, $\cos\theta \cdot \dfrac{d\theta}{dt} = \dfrac{0.4}{\sqrt{2}}$

$$\therefore \quad \frac{d\theta}{dt} = \frac{0.4}{\sqrt{2}}\frac{1}{\sqrt{1-\sin^2\theta}} = \frac{0.4}{\sqrt{2}} \cdot \frac{1}{\sqrt{1-\frac{1.4\times1.4}{2}}}\, rad/\sec = \frac{0.4}{\sqrt{0.04}}\, rad/\sec$$

$$\frac{d\theta}{dt} = 2 rad/\sec$$

2.(8) $A_1V_1 = A_2V_2; \quad \rho_{air} = 1 kg/m^3$

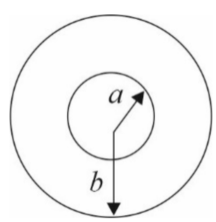
$$P_a - \rho_{\text{water}}\, gh + \frac{1}{2}\rho_{\text{air}}(4V_2)^2 = P_a + \frac{1}{2}\rho_{\text{air}}(V_2)^2$$

$$\Rightarrow \quad \frac{15}{2}\rho_{\text{air}} V_2^2 = \rho_{\text{water}}\, gh \quad \Rightarrow \quad V_2^2 = \frac{2\rho_{\text{water}}\, gh}{15\rho_{\text{air}}} \quad \Rightarrow \quad V_2 = 50\, m/s$$

$$Q = A_2V_2 = 16\times10^{-4}\times 50 = 80\times10^{-3} m^3/s$$

3.(4) $\dfrac{q}{C} = \dfrac{-dq}{dt}R$

$$\Rightarrow \quad \int_{q_0}^{q_0/n}\frac{dq}{q} = \frac{-1}{RC}\int_0^{\tau} dt \quad \Rightarrow \quad \frac{\tau}{RC} = \log_e n$$



$$\Rightarrow \quad R = \frac{\tau}{C\log_e n} \quad \Rightarrow \quad \rho\frac{b-a}{4\pi ab} = \frac{\tau}{C\log_e n}$$

$$\therefore \quad \rho = \frac{4\pi ab\tau}{(b-a)C\log_e n} \quad \therefore \quad x = 4$$

4.(6) $\frac{1}{2}KR^2 + mg2R = \frac{1}{2}mV^2$

$\Rightarrow \quad mgR + 4mgR = mv^2 \quad \Rightarrow \quad v = \sqrt{5gR}$

$N = \frac{mV^2}{R} + mg = 5mg + mg \quad \Rightarrow \quad N = 6mg$

5.(2) $\alpha = \frac{\tau_C}{I_C} = \frac{mg.x}{mb^2 - m(b-x)^2 + mx^2} \quad \Rightarrow \quad \alpha = \frac{gx}{2bx} \quad \Rightarrow \quad \alpha = \frac{g}{2b}$

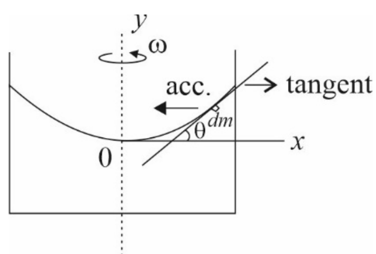
6.(3) $N_V = 0 = B - A(V-1)^2$ for V_{max} and V_{min}

$V_{min} = 1 - \sqrt{\frac{B}{A}}; \qquad V_{max} = 1 + \sqrt{\frac{B}{A}}$

$$V_{avg} = \frac{\int_{V_{min}}^{V_{max}} \left[B - A(V-1)^2\right] T\, dV}{N_0} = \frac{4}{3}\frac{BT_0}{N_0}\left[\frac{B}{A}\right]^{1/2}$$

<u>ONE OR MORE THAN ONE CHOICE</u>

7.(ABC)



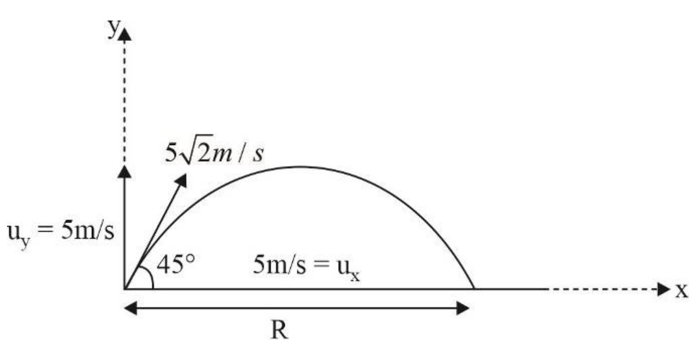
$acc. = \omega^2 x$

$\tan\theta = \frac{acc.}{g}$

$\frac{dy}{dx} = \frac{\omega^2 x}{g} \quad \Rightarrow \quad y = \frac{\omega^2}{g}\int x dx$

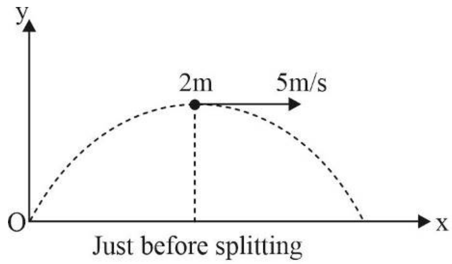
$y = \frac{\omega^2}{2g}x^2 \quad \Rightarrow$ Parabolloid if parabolic curve is rotated about y-axis.

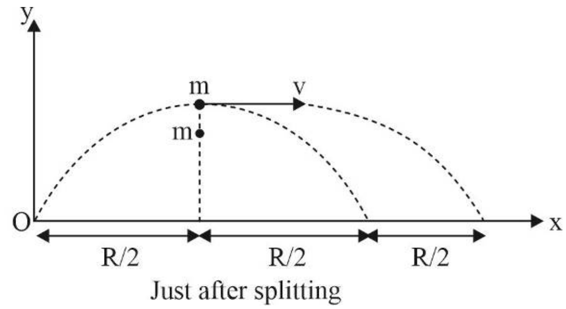
8.(AC)



Range $R = \frac{2u_x u_y}{g} = \frac{2\times5\times5}{10} = 5m$

Time of flight $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1\text{sec}$





$\because$ Time of motion of one part falling vertically downwards is $= 0.5\sec = \dfrac{T}{2}$

$\Rightarrow$ Time of motion of another part, $t = \dfrac{T}{2} = 0.5\sec$

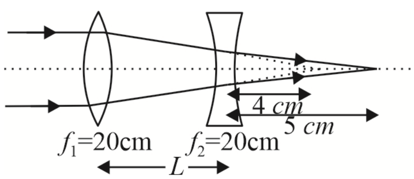
From momentum conservation $\Rightarrow P_i = P_f$

$2m \times 5 = m \times v \quad \Rightarrow \quad v = 10\,m/s$

Displacement of other part in 0.5 sec in horizontal direction $= v\dfrac{T}{2} = 10 \times 0.5 = 5\,m = R$

$R = 5m = x \quad \Rightarrow \quad t = 0.5\sec$

9.(AC)



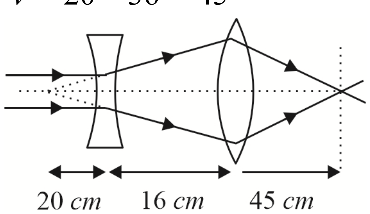
$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{5} - \frac{1}{u_2} = \frac{1}{-20}$$

$\therefore \quad u_2 = +4\ cm \quad \therefore \quad L = 16\ cm$

After interchanging the lens

For convex lens $\dfrac{1}{v} - \dfrac{1}{-36} = \dfrac{1}{20}$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{36} = \frac{1}{45} \quad \Rightarrow \quad v = 45\,cm$$



10.(ABC)

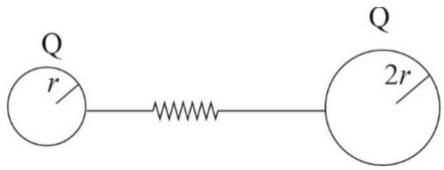
$$\lambda_{\min} = \frac{hc}{eV} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 20 \times 1000} m \quad \Rightarrow \quad \lambda_{\min} = 0.62\,\text{Å}$$

Max loss in KE = $19.9\,keV$ = Max energy of X-rays

L_α X-ray may be emitted.

11.(AD)

(A) $$I=\frac{V}{R}=\left(\frac{kQ}{r}-\frac{kQ}{2r}\right)\frac{1}{R}$$



$$I=\frac{Q}{8\pi\varepsilon_0 rR}$$

(D) Current at time t

$$I=\left(\frac{Q-q}{r}-\frac{Q+q}{2r}\right)\frac{1}{4\pi\varepsilon_0 R} \quad\Rightarrow\quad I=\frac{1}{8\pi\varepsilon_0 rR}(Q-3q)$$

$$\Rightarrow\quad \frac{dI}{dt}=\frac{-3}{8\pi\varepsilon_0 rR}I \quad\Rightarrow\quad \int_{I_0}^{I}\frac{dI}{I}=\frac{-3}{8\pi\varepsilon_0 rR}\int dt \quad\Rightarrow\quad \log_e\left|\frac{I}{I_0}\right|=-\frac{3t}{8\pi\varepsilon_0 rR}$$

$$I=I_0 e^{-\frac{3t}{8\pi\varepsilon_0 rR}} \quad\Rightarrow\quad I=\frac{I_0}{2} \quad\Rightarrow\quad e^{\frac{3t}{8\pi\varepsilon_0 rR}}=2 \quad\Rightarrow\quad t=\frac{8\pi\varepsilon_0 rR}{3}\ln 2$$

12.(C) Rate of decay $\propto a$

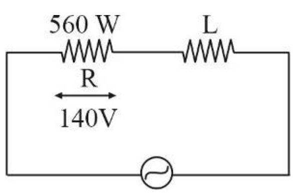
$$\Rightarrow\quad -\frac{da}{dt}=Ka \quad\Rightarrow\quad -\int_{a_0}^{a}\frac{da}{a}=K\int_0^t dt \quad\Rightarrow\quad \log_e\left|\frac{a}{a_0}\right|=-Kt \quad\Rightarrow\quad a=a_0e^{-Kt}$$

At t_0, $\frac{a_0}{2}=a_0e^{-Kt_0} \quad\Rightarrow\quad e^{Kt_0}=2 \quad\Rightarrow\quad Kt_0=\ln 2 \quad\therefore\quad K=\frac{\ln 2}{t_0}$

Again, $\frac{dv}{dt}=a_0e^{-Kt} \quad\Rightarrow\quad v=\frac{a_0}{K}\left(1-e^{-Kt}\right) \quad\Rightarrow\quad v_T=\frac{a_0}{K}=\frac{a_0t_0}{\ln 2}$

NUMERICAL VALUE TYPE

13.(0.5)



$220V$, $\omega=60$ rad/sec

$$P=i_{rms}V_R\Rightarrow i_{rms}=\frac{560}{140}=4A$$

$$V_R^2+V_L^2=V_{rms}^2$$

$$V_L^2=(220)^2-(140)^2 \Rightarrow V_L=120\sqrt{2}$$

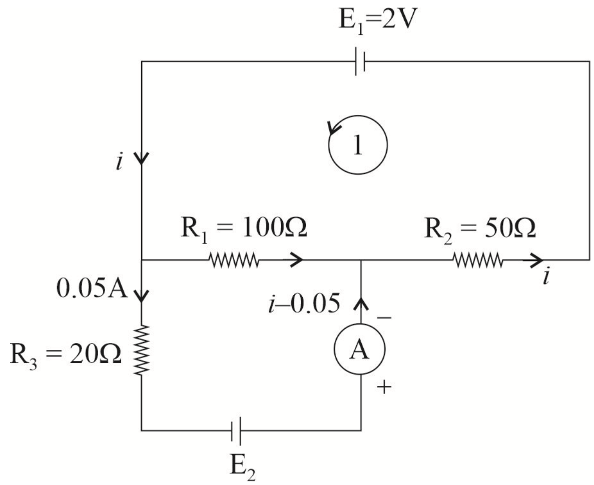
Also, $V_L=i_{rms}\cdot X_L$

$$120\sqrt{2}=4\omega L \quad\Rightarrow\quad L=\frac{120\sqrt{2}}{4\times 60} \quad\Rightarrow\quad L=\frac{\sqrt{2}}{2}=\sqrt{\frac{1}{2}}$$

14.(1.33)

Loop (1)

$$2-100(i-0.05)-50i=0 \quad \Rightarrow \quad 7=150i \quad \Rightarrow \quad i=\frac{7}{150}A$$



Loop (2)

$$-20\times0.05+E_2+\left(\frac{7}{150}-0.05\right)100=0 \quad \Rightarrow \quad -1+E_2+\frac{14}{3}-5=0$$

$$E_2=6-\frac{14}{3}=\frac{4}{3}V=1.33V$$

15.(3.63)

Zero of vernier will lie close to $78-6=72$ division

Reading $=72\times0.5+6\times0.05=36.3\,mm=3.63\,cm$

16.(4) ω will be same for both stars

$$\frac{V_{areal_1}}{V_{areal_2}}=\frac{\frac{1}{2}\omega r_1^2}{\frac{1}{2}\omega r_2^2} \quad \Rightarrow \quad \frac{V_{areal_1}}{V_{areal_2}}=\frac{r_1^2}{r_2^2}$$

Also, $M_1r_1=M_2r_2$ (C.O.M.)

$$\frac{r_1}{r_2}=\frac{M_2}{M_1}$$

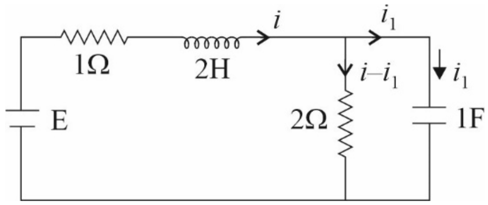
$$\frac{V_{areal_1}}{V_{areal_2}}=\frac{M_2^2}{M_1^2}=\left(\frac{6M}{3M}\right)^2=4$$

17.(3) $i_1=2e^{-t}$

Initial current $=i_{1,\,0}=2A$

$$\frac{dq}{dt}=2e^{-t} \quad \Rightarrow \quad \int_{q_0}^{q}dq=2\int_0^t e^{-t}dt$$

$$\Rightarrow \quad q - q_0 = -2\left(e^{-t} - 1\right) \quad \Rightarrow \quad q = q_0 + 2\left(1 - e^{-t}\right)$$



$$E - i - 2\frac{di}{dt} - \left(i - i_1\right)2 = 0$$

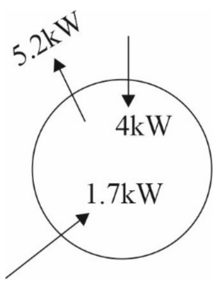
$$2\left(i - i_1\right) - \frac{q}{1} = 0 \quad \Rightarrow \quad i - i_1 = \frac{q}{2} \quad \Rightarrow \quad \frac{di}{dt} - \frac{di_1}{dt} = \frac{i_1}{2}$$

$$\frac{di}{dt} = -2e^{-t} + e^{-t} = -e^{-t} \quad \Rightarrow \quad \int_0^{E/3} di = \int_0^{\infty} e^{-t} dt$$

$$\Rightarrow \quad \frac{E}{3} = \left|\frac{-1}{e^t}\right|_0^{\infty} \quad \Rightarrow \quad \frac{E}{3} = 1 \quad \therefore \quad E = 3 \text{ volt}$$

18.(1000)

Net rate of heat loss $= 4kW + 1.7kW - 5.2kW$



$$\Rightarrow \quad C\frac{\Delta T}{\Delta t} = 0.5\,kW \quad \Rightarrow \quad C \times 0.5 = 500 \quad \therefore \quad C = 1000$$

CHEMISTRY

SINGLE DIGIT INTEGER TYPE

1.(3) The difference between I[st] and 2[nd] I.E of element P is maximum among all. So, P is alkali metal and n is given less than 6. Hence atomic no P is 3.

2.(5) Only polar molecules can be attracted by a charged comb. So, except S_2, CS_2 and $p-C_6H_4(Cl)_2$, all can be attracted in electric field.

3.(4) $\text{eq.wt. of } K_2Cr_2O_7 = \frac{\text{molar mass}}{6}$

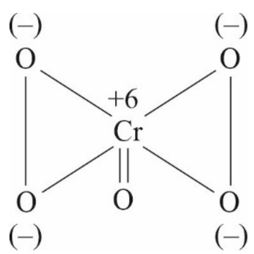
$\text{eq. wt. of } KMnO_4 = \frac{\text{molar mass}}{5}$

$\text{eq. of } Na_2S_2O_3 = \text{eq. of } I_2 \text{ liberated} = \text{eq. of } KMnO_4 + \text{eq. of } K_2Cr_2O_7$

Or N × 1 = 0.02 × 5 + 0.05 × 6 ⇒ N = 0.4 or M = 0.4 ⇒ $M = 4 \times 10^{-1}$

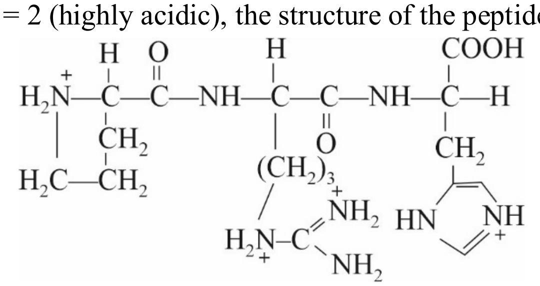
4.(4) $CrO_4^{2-} \underset{OH^-}{\overset{H^+}{\rightleftharpoons}} Cr_2O_7^{2-}$

$Cr_2O_7^{2-} + H_2O + H^+ \longrightarrow \underset{\substack{\text{Chromium peroxide} \\ \text{(blue)}}}{CrO_5}$



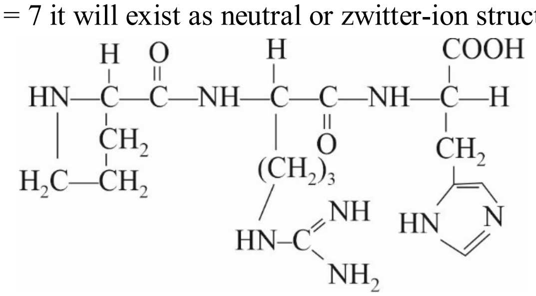
Which are in –1 oxidation state.

5.(5) At pH = 2 (highly acidic), the structure of the peptide will be as shown below.



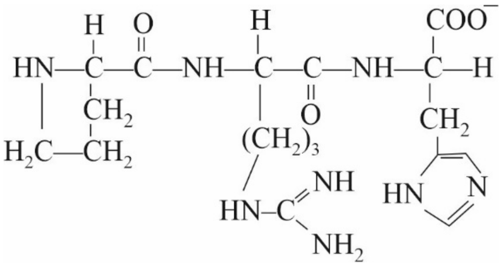
So, $|Z_1| = 4$

At pH = 7 it will exist as neutral or zwitter-ion structure as shown below:



So, according to this condition, $|Z_2| = 0$

At pH = 11 (highly basic), the given peptide will exist as:

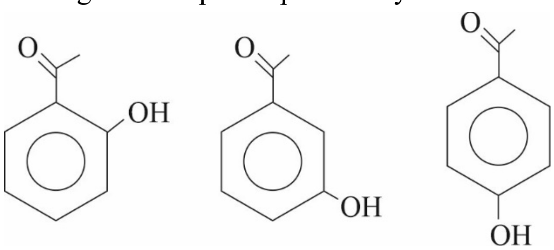


So, $|Z_3| = |-1| = 1$

$|Z_1| + |Z_2| + |Z_3| = 4 + 0 + 1 = 5$

6.(3) 3 (considering that all isomers satisfy the given conditions)

Since given compound produces yellow colour with NaOI solution.



So, Total isomeric structures = 3

ONE OR MORE THAN CHOICE

7.(ABC)

Diamagnetic (all electrons paired) are repelled i.e., magnetized in opposite direction in magnetic field, so such substance are deflected upwards in magnetic field.

Diamagnetic substances are $K_4\left[Fe(CN)_6\right], NaCl(s)$

Paramagnetic substances (at least one unpaired electron) are attracted in magnetic field, so such substances are deflected downwards.

$O_2^-(g): \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\sigma_{2p_z^2}\pi_{2p_x^2} = \pi_{2p_y^2}\ \pi^*_{2p_x^2} = \pi^*_{2p_y^1}$

Since O_2^- molecules has one unpaired electrons in antibonding π^*. MO.

So KO_2 is paramagnetic. And $B_2(l)$ is also paramagnetic.

8.(A) The order of the above reaction is one (S_N1) because carbocation intermediate is stablised by resonance.

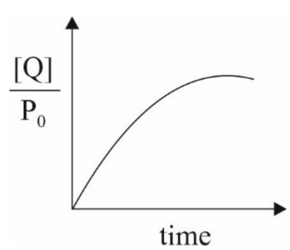
So, for a 1st order reaction

$$\frac{P}{P_0} = e^{-kt}$$

$$\ln\left[\frac{P}{P_0}\right] = -kt$$

$$\left(1 - \frac{P}{P_0}\right) = \left(1 - e^{-kt}\right)$$

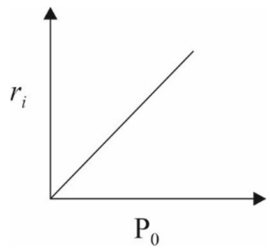
$$\frac{P_0 - P}{P_0} = \left(1 - e^{-kt}\right)$$



or $\frac{[Q]}{P_0} = \left(1 - e^{-kt}\right)$

So, graph between $\frac{(Q)}{P_0}$ Vs. t, should be as shown in figure (1), so, option (C) is incorrect

For a first order reaction



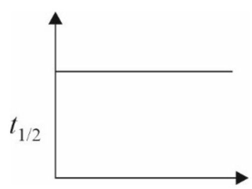
$r = k[P]^1$

At $t = 0, r_i = k[P_0]$

So, initial rate of reaction is directly proportional to initial conc. of P.

Hence option B is also incorrect.

For a first order reaction



$t_{1/2} = \frac{\ln 2}{k}[P_0]^0$

So, $t_{1/2}$ is independent of initial conc. of reactant. So, option (A) is correct.

9.(ABD)

Fact (Thermite process reaction not occurs in blast furnance)

10.(AB)

Fact

$2Al + 2KOH + 2H_2O \longrightarrow 2KAlO_2 + 3H_2$

$SiO_2 + HF \rightarrow H_2SiF_6 + H_2O$

11.(ABC)

pK_b value of these bases are respectively 13.2, 3.35, 3.05, 8.77

12.(AB)

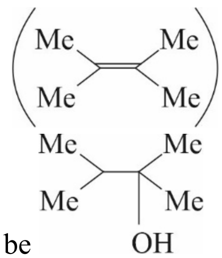
On proceeding reverse, following observations are made:

Acetone obtained from (G) suggests that (G) is calcium acetate, $(CH_3COO)_2Ca$.

Therefore, compound (D) is acetone $(Me_2C=O)$

Thus, compound (F) would be isopropyl alcohol (Me_2CHOH).

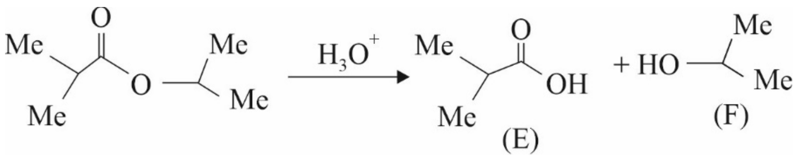
Since, (D) is obtained from (C), (C) would be



Would be

Structure of (D) and (F) suggest that ester (A)

Should be



NUMERICAL VALUE TYPE

13.(5) $HA + OH^- \rightleftharpoons A^- + H_2O$

	HA	OH^-	A^-
	a millimole	36.12×0.1 Millimole	0
Equ. Point	$\simeq 0$	$\simeq 0$	3.612 millimole

$A^- + H^+ \rightleftharpoons HA$

	A^-	H^+	HA
	3.612mmole	18.06×0.1 m mole	0
Final	1.806mmole	$\simeq 0$	1.806 m mole

$\therefore\ pH = pK_a + \log\frac{[A^-]_0}{[HA]_0} = 5$

As $[A] = [A]$, So $pH = pK_a$

14.(54) $P = X_2.P^\circ$

$20 = \frac{180/18}{\frac{6}{M} + \frac{180}{18}} \times P^\circ$ ...(i)

and $20.02 = \frac{11}{\frac{6}{M} + 11} \times P^\circ$ ...(ii)

Hence, M = 54

15.(19) $P.E. = -\frac{Kq_1q_2}{r}$

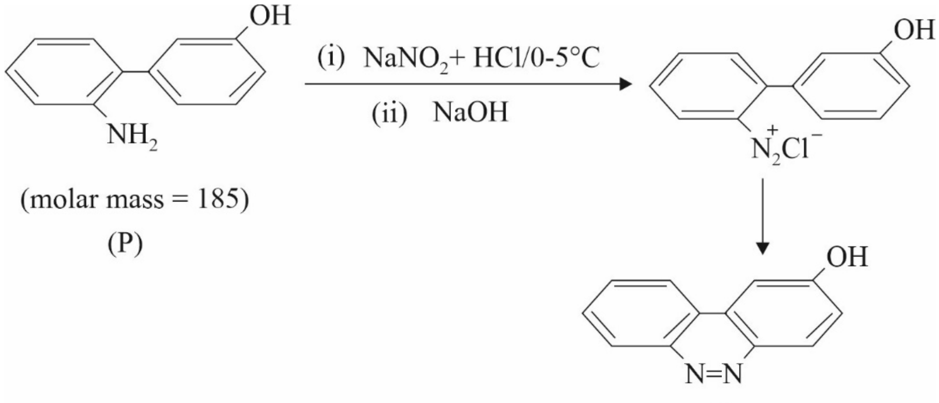
P.E. of one H-atom in ground state $= -\frac{K(e)(e)}{r}$

P.E. $= \frac{-9\times10^9 \times (1.6\times10^{-19})^2}{0.529\times10^{-10}} = -43.5538\times10^{-19}\,J/atom$

So, P.E. of two H-atoms (or one molecule of H_2)

$= -2\times43.5538\times10^{-19} = -87.106\times10^{-19}\,J/molecule$

16. (14.70)



Moles of $P = \frac{18.5}{185} = 0.1 =$ moles of Q.

So, mass of (Q) = 0.1 × 196 × 0.75 = 14.70 gm.

17.(169) $\Delta S^0 = S_{CH_4} - \left(S_{grap} + 2 \times S_{H_2}\right) = 186.2 - \left(6.0 + 2 \times 130.6\right) = -81\ J/K$

Now, $\Delta G° = \Delta H° - T.\Delta S° = -T.\Delta S_{univ}$

or, $\left(-75 \times 10^3\right) - 300 \times \left(-81\right) = -300 \times \Delta S_{univ} \quad \Rightarrow \quad \Delta S_{univ} = 169 J/K$

18.(1) $M^{2+} + H_2S \rightleftharpoons MS(S) + 2H^+$

For ppt. of MS(s), $Q < k_{eq}$

or, $\frac{\left[H^+\right]}{\left[M^{2+}\right]\left[H_2S\right]} < \frac{K_{a_1}.K_{a_2}}{K_{sp}}$

or, $\frac{\left[H^+\right]^2}{0.04 \times 0.1} < \frac{10^{-7} \times 1.5 \times 10^{-13}}{6 \times 10^{-21}}$

$\therefore \quad \left[H^+\right]_{max} = 0.1M \Rightarrow P^H_{min} = 1.0$

MATHEMATICS

SINGLE DIGIT INTEGER TYPE

1.(6) $t_r = \dfrac{a_r^3}{1-3a_r+3a_r^2} = \dfrac{a_r^3}{a_r^3+(1-a_r)^3} = \dfrac{r^3}{r^3+(101-r)^3}$

$\Rightarrow \quad t_r + t_{101-r} = 1 \quad \therefore \quad S_{100} = \sum_{r=1}^{50}(t_r + t_{101-r}) = 50$

$S_{101} = S_{100} + t_{101} = 50 + 1 = 51$

Sum of digits = 5 + 1 = 6

2.(7) Let $P(E_1) = a,\ P(E_2) = b$ and $P(E_3) = c$

$3a(1-b)(1-c) = (1-a)\,b\,(1-c) = 9\,(1-a)(1-b)\,c = 3\,(1-a)(1-b)(1-c)$

$\dfrac{3a}{1-a} = \dfrac{b}{1-b} = \dfrac{9c}{1-c} = 3 \quad \Rightarrow \quad a = \dfrac{1}{2}, b = \dfrac{3}{4}, c = \dfrac{1}{4}$

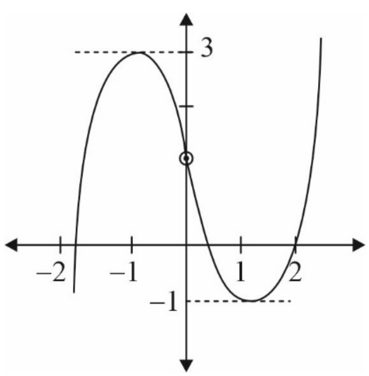
Now, $\begin{vmatrix} 1/2 & 3/4 & 1/4 \\ 3/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{vmatrix} = \dfrac{1}{64}\begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\dfrac{9}{32}$

$\therefore \quad \dfrac{a}{b} = \dfrac{9}{32} \quad \Rightarrow \quad a + b = 41$

$(a+b+1)/6 = 7$

3.(7) $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$

$f(1) = -1,\ f(-1) = 3$



Where $-2 < x_1 < -1,\ 0 < x_2 < 1,\ 1 < x_3 < 2$

$\therefore \quad f(f(x)) = 0 \quad \Rightarrow \quad f(x) = x_1$ has 1 solution

$f(x) = x_2$ has 3 solutions and $f(x) = x_3$ has 3 solutions.

$\therefore$ Number of solutions = 7

4.(2) $a_{ij} = 0\ \forall\ i \neq j$ and $a_{ij} = (n-1)^2 + i,\ i = j$

Sum of all the element of

$$=(2n-1)(n-1)^2+(2n-1)n=2n^3-3n^2+3n-1=n^3+(n-1)^3$$

So, $T_n=(-1)^n\left[n^3+(n-1)^3\right]=(-1)^n n^3-(-1)^{n-1}(n-1)^3=V_n-V_{n-1}$

$$\Rightarrow \quad \sum_{n=1}^{102}T_n=\sum_{n=1}^{102}(V_n-V_{n-1})=V_{102}-V_0=(102)^3 \quad \Rightarrow \quad \left[\frac{\sum_{n=1}^{102}T_n}{520200}\right]=2$$

5.(0) $I_1=\int_{-4}^{-5}e^{(x+5)^2}dx=(-5+4)\int_0^1 e^{((-5+4)x-4+5)^2}dx$

$I_1=-\int_0^1 e^{(x-1)^2}dx$ …… (i)

Again let $I_2=\int_{1/3}^{2/3}e^{9\left(x-\frac{2}{3}\right)^2}dx$

$I_2=\left(\frac{2}{3}-\frac{1}{3}\right)\int_0^1 e^{9\left[\left(\frac{2}{3}-\frac{1}{2}\right)x+\frac{1}{3}-\frac{2}{3}\right]^2}dx=\frac{1}{3}\int_0^1 e^{(x-1)^2}dx=\frac{1}{3}(-I_1)$ …… (ii)

Then $I=I_1+3I_2=I_1+3\left(-\frac{I_1}{3}\right)=I_1-I_1 \quad \Rightarrow \quad I=0$

$$\int_{-4}^{-5}e^{(x+5)^2}dx+3\int_{1/3}^{2/3}e^{9\left(x-\frac{2}{3}\right)^2}dx=0$$

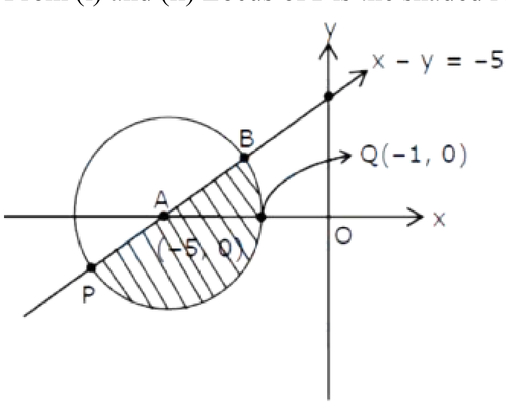
6.(8) Given, $|z+5|\le 4$

$\Rightarrow \quad (x+5)^2+y^2\le 16$ …… (i)

Also, $z(1+i)+\bar{z}(1-i)\ge -10$

$\Rightarrow \quad x-y\ge -5$ …… (ii)

From (i) and (ii) Locus of z is the shaded region in the diagram.



$|z+1|$ represents distance of 'z' from $Q(-1, 0)$ Clearly 'P' is the required position 'z' when $|z+1|$ is maximum.

$\therefore \quad P \equiv \left(-5-2\sqrt{2}, -2\sqrt{2}\right) \quad \therefore \quad (PQ)^2_{\max} = 32+16\sqrt{2}$

$\Rightarrow \quad a = 32; \quad b = 16$

$\Rightarrow \quad 32x^2 = 4(16y^2 + 32\times16) \quad \Rightarrow \quad 8x^2 - 16y^2 = 32\times16$

$\Rightarrow \quad \dfrac{x^2}{4\times16} - \dfrac{y^2}{32} = 1 \quad \Rightarrow \quad \dfrac{x^2}{8^2} - \dfrac{y^2}{32} = 1$

Length of LR is $\dfrac{2b^2}{a} = \dfrac{2\times32}{8} = 8$

ONE OR MORE THAN ONE CHOICE

7.(ACD)

Apply L hospital rule for both limits

$\ell_1 = \dfrac{4}{3}; \qquad \ell_2 = 1$

$3\ell_1 + 4\ell_2 = 8$

8.(ABCD)

Given $\quad x^2 + y^2 - 2x - 2y + 1 = 0 \qquad \ldots\ldots$ (i)

$y = mx \qquad \ldots\ldots$ (ii)

Solving $\left(1+m^2\right)x^2 - 2(m+1)x + 1 = 0$

$$L.C = |x_1 - x_2|\sqrt{1+m^2} = \frac{\sqrt{4(m+1)^2 - 4\left(m^2+1\right)}}{\left(1+m^2\right)}\sqrt{1+m^2} = \sqrt{\frac{8m}{1+m^2}}$$

$$\sqrt{\frac{8m}{1+m^2}} = \frac{4}{\sqrt{5}} \Rightarrow 16m^2 + 16 = 40m \Rightarrow 2m^2 - 5m + 2 = 0 \Rightarrow m = 2, \frac{1}{2}$$

$$f(m) = \text{Area} = \frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}\times\sqrt{\frac{8m}{1+m^2}}\times\frac{1}{\sqrt{1+m^2}}$$

$\Rightarrow \quad f'(m) = 0 \quad \Rightarrow \quad m^2 = 1/3$

9.(ABD)

Graph is symmetrical about (4, 0)

$\Rightarrow \quad f(4+x) = -f(4-x) \quad \Rightarrow \quad f(x) = -f(8-x)$

Now let $f(x) = 2020$ then $f(8-x) = -2020$

$\Rightarrow \quad f^{-1}(2020) + f^{-1}(-2020) = 8$

$\Rightarrow \quad$ Option A is correct and

$$\int_{-2020}^{4} f(x)dx = -\int_{4}^{2028} f(x)\,dx \quad \therefore \quad \int_{-2020}^{2028} f(x)\,dx = 0$$

Also $D=\left(f'(10)\right)^2+4f'(10)>0$

$\therefore \quad f'(-100)>0 \quad \Rightarrow \quad f'(10)\ge 0$

$\Rightarrow \quad x^2-f'(10)\,x-f'(10)=0$ has real roots

$\Rightarrow$ Option C is not correct

As $\quad f'(4+x)=f'(4-x)$

$\Rightarrow \quad f'(10)=f'(-2)=20$

10.(AD)

$L_1: x=y=z$

$L_2: \dfrac{x-1}{1}=\dfrac{y+1}{-1}=\dfrac{z}{-1}$

Shortest distance $=\dfrac{1}{\sqrt{2}}$

Equation of plane containing line L_2 and parallel to L_1

$y-z+1=0$

Distance of origin from this plane $=\dfrac{1}{\sqrt{2}}$

11.(AD)

The vector equation of the line through P and parallel to $\vec{A}$ is

$\vec{r}=\left(5\hat{i}+7\hat{j}-2\hat{k}\right)+t\left(3\hat{i}-\hat{j}+\hat{k}\right)=(5+3t)\hat{i}+(7-t)\hat{j}-(2-t)\hat{k}$ …… (i)

The vector equation of the line through Q and parallel to $\vec{B}$ is

$\vec{r}=\left(-3i+3\hat{j}+6\hat{k}\right)+t'\left(-3\hat{i}+2\hat{j}+4\hat{k}\right)=-(3+3t')\hat{i}+(3+2t')\hat{j}+(6+4t')\hat{k}$ …… (ii)

Let the third vector $2\hat{i}+7\hat{j}-5\hat{k}$ intersects line (i) and (ii) in L and M.

So, $m\left(2\hat{i}+7\hat{j}-5\hat{k}\right)=\overrightarrow{LM}=PV$ of $M-PV$ of L

$=(-3-3t'-5-3t)\hat{i}+(3+2t'-7+t)\hat{j}+(6+4t'+2-t)\hat{k}$

Thus, $-8-3t-3t'=2m,\ -4+2t'+t=7m,\ 8+4t'-t=-5m$

Solving these we get

$m=-1,\ t=-1,\ t'=-1$

Thus, the PV of L is $2\hat{i}+8\hat{j}-3\hat{k}$ and the PV of M is $\hat{j}+2\hat{k}$

12.(D) Given $[(1-x)(1+x+x^2)]^n=\sum_{k=0}^{n} a_k x^k \dfrac{(1-x)^{3n}}{(1-x)^{2k}}$

$$\Rightarrow \quad \left[\frac{1+x+x^2}{(1-x)^2}\right]^n=\sum_{k=0}^{n} a_k \frac{x^k}{(1-x)^{2k}}$$

$$\Rightarrow \quad \left[\frac{(1-x)^2+3x}{(1-x)^2}\right]^n = \sum_{k=0}^{n} a_k \left(\frac{x}{(1-x)^2}\right)^k$$

$$\Rightarrow \quad \left[1+\frac{3x}{(1-x)^2}\right]^n = \sum_{k=0}^{n} a_k \left(\frac{x}{(1-x)^2}\right)^k$$

Put $\frac{x}{(1-x)^2} = t \quad \Rightarrow \quad (1+3t)^n = \sum_{k=0}^{n} a_k\, t^k$

Now $3a_{k-1} + a_k = 3\left(\text{coeff of } t^{k-1}\right) + \text{coeff of } t^k$

$$= 3\cdot 3^{k-1}\ {}^nC_{k-1} + 3^k \cdot {}^nC_k = 3^k\{{}^nC_{k-1} + {}^nC_k\} = 3^k \cdot {}^{n+1}C_k$$

NUMERICAL VALUE TYPE

13.(673)

Let the angles are $x°, y°, z°$

$x + y + z = 180; \qquad x, y, z \in I$

Case-I : All angles same 60,60,60 ...(i)

Case-II : Two angles same $\left.\begin{matrix}1,1,178\\2,2,176\\89,89,2\end{matrix}\right\}(88)$

Case-III All the angles are different

$(88+85+....+)+(86+83....) = 1335+1276 = 2611$

$1+88+2611 = 2700$

Number of zeros will be $\left[\frac{2700}{5}\right]+\left[\frac{2700}{5^2}\right]+\left[\frac{2700}{125}\right]+\left[\frac{2700}{625}\right] = 540+108+21+4 = 673$

14.(7) $\left(1+x+\frac{1}{x^2}+\frac{1}{x^3}\right)^{10} = \frac{\left(1+x+x^3+x^4\right)^{10}}{x^{30}}$

coefficient of x^{30} in $\left(1+x+x^3+x^4\right)^{10}$

$= {}^{10}C_{10}\ {}^{10}C_0 + {}^{10}C_9\ {}^{10}C_3 + {}^{10}C_8\ {}^{10}C_6 + {}^{10}C_7\ {}^{10}C_9$ = 1 + 1200 + 9450 + 1200

Last digit of $(11853)^{11851} = 7$

15.(14) 9 + 8 + 6 + 5 + 4 = 32

$\Rightarrow$ Set $P(A, B, C, D, E, F) = (2, 3, 4, 5, 6, 8, 9)$

$n(S) = {}^7C_5 \cdot 5!$

For total number of numbers divisible by 4, the digits in unit and tens place can be filled in (3 × 2 + 2 × 2 + 1 + 1) ways

So, total number of numbers thus formed = 60 × 12 = 720

16.(0) $\dfrac{\sin^2\left(x-\dfrac{\pi}{4}\right)}{\cos 2x}=\dfrac{\dfrac{1}{2}(\sin x-\cos x)^2}{\cos^2 x-\sin^2 x}=-\dfrac{1}{2}\tan\left(x-\dfrac{\pi}{4}\right)$

Given equation reduces to $3^{\tan\left(x-\frac{\pi}{4}\right)}-2\left(\dfrac{1}{9}\right)^{-\frac{1}{2}\tan\left(x-\frac{\pi}{4}\right)}+1=0$

$\Rightarrow \quad 3^{\tan\left(x-\frac{\pi}{4}\right)}=1 \quad \Rightarrow \quad x=n\pi+\dfrac{\pi}{4},\ n\in z$

Which is not possible as $\cos 2x\neq 0$

17.(42) $I_n=\int\limits_0^{\infty} e^{-x}(\sin x)^n\,dx$

$=\left[\sin^n x\left(-e^{-x}\right)\right]_0^{\infty}+\int\limits_0^{\infty} n\sin^{n-1}x\cos x e^{-x}dx=0+n\int\limits_0^{\infty}\left(\sin^{n-1}x\cos x\right)e^{-x}dx$

$=n\left[\left(\sin^{n-1}x\cos x\right)\left(-e^{-x}\right)\right]_0^{\infty}-n\int\limits_0^{\infty}\left\{-\sin^n x+(n-1)\sin^{n-2}x\cos^2 x\right\}\left(-e^{-x}\right)dx$

$=0+n\int\limits_0^{\infty} e^{-x}\left\{-\sin^n x+(n-1)\sin^{n-2}x\left(1-\sin^2 x\right)\right\}dx$

$=n\int\limits_0^{\infty} e^{-x}\left\{(n-1)\sin^{n-2}x-n\sin^n x\right\}dx=n(n-1)I_{n-2}-n^2 I_n$

We have, $\left(1+n^2\right)I_n=n(n-1)l_{n-2}$ then $\dfrac{l_n}{l_{n-2}}=\dfrac{n(n-1)}{n^2+1}$

18.(6) Let $\tan x=a$, $\tan y=b$, $\tan z=c$

Given system of equation is equal to

$a+b+c=6-\dfrac{1}{a}-\dfrac{1}{b}-\dfrac{1}{c}$

$a^2+b^2+c^2=6-\dfrac{1}{a^2}-\dfrac{1}{b^2}-\dfrac{1}{c^2}$

$a^3+b^3+c^3=6-\dfrac{1}{a^3}-\dfrac{1}{b^3}-\dfrac{1}{c^3}$

For second equation we complete squares to get

$\left(a-\dfrac{1}{a}\right)^2+\left(b-\dfrac{1}{b}\right)^2+\left(c-\dfrac{1}{c}\right)^2=0 \quad \Rightarrow \quad a=b=c=\pm 1$

Now rearranging 3rd equation and adding 3-time first equation, we get

$a^3+b^3+c^3+\dfrac{1}{a^3}+\dfrac{1}{b^3}+\dfrac{1}{c^3}+3\left(a+\dfrac{1}{a}+b+\dfrac{1}{b}+c+\dfrac{1}{c}\right)=6+18$

$$\left(a+\frac{1}{a}\right)^3+\left(b+\frac{1}{b}\right)^3+\left(c+\frac{1}{c}\right)^3=24$$

Therefore $a=b=c=1$

Hence, $\left[\frac{\tan(x)}{\tan(y)}+\frac{\tan(y)}{\tan(z)}+\frac{\tan(z)}{\tan(x)}+3\tan(x)\tan(y)\tan(z)\right]=6$